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Design Equations for Broadside and Edgewise Stripline Couplers

Designers wishing to split RF power at a desired proportion between two lines may do so by designing a four port stripline coupler device. For this discussion a coupler shall consist of two parallel stripline transmission lines having equal widths, equal distances from ground planes and uniform spacing between them running for a distance of 1/4 of a wavelength at the design frequency. Connections to both ends of the two lines are arranged to be out of the coupled region and to have the designed characteristic impedance (Z_0). Ports 1 and 2 are on the driven line with the coupled line's port 3 adjacent to port 2 and port 4 adjacent to port 1. Maximum coupling is at the design frequency and is expressed as the power ratio of port 4 to port 1 expressed in decibel (dB) units. Ideally all power incident at port 1 splits between ports 2 and 4 with none going to port 3. Since a dB unit is $-10 \log_{10}$ (power ratio) a higher coupling coefficient is represented by a lower positive dB value. 3 dB represents approximately an equal power split between ports 2 and 4, while 10 dB represents a 9 to 1 split or lower coupling to the other line.

As a convenience for coupled stripline device design we show formulas on the next page relating the characteristic impedance, Z_0, and the coupling coefficient in decibels (dB) to line spacing and line width. The stripline broadside coupler usually used for high coupling is built from three circuit boards forming two signal layers between ground planes as shown in Figure 1. The stripline edgewise coupler usually used for lower coupling needs only two boards with a single signal layer as shown in Figure 2. We do not cover here the more versatile offset broadside coupler design.

Certain relationships among D, Z_0 and V on page 2 are common to both broadside and edgewise couplers as shown in section A. Note that for the broadside case in section B the symbols s and w represent the ratios of line spacing and line width to the total ground plane spacing. The total ground plane spacing is the sum of the thicknesses of the three boards. Note that for the edgewise case in section C the symbols w, s and b represent dimensional unit values for width, spacing and ground plane spacing respectively.

Several assumptions are made for both cases:

1. Thickness of the conductors is considered negligible.
2. The coupled lines have equal width.
3. Distance to ground plane on either side of the coupled lines is equal.
4. Dielectric material completely fills the space between ground planes not occupied by conductor, a condition achieved by bonding.
5. All layers of dielectric material have the same relative permittivity (ε_r).

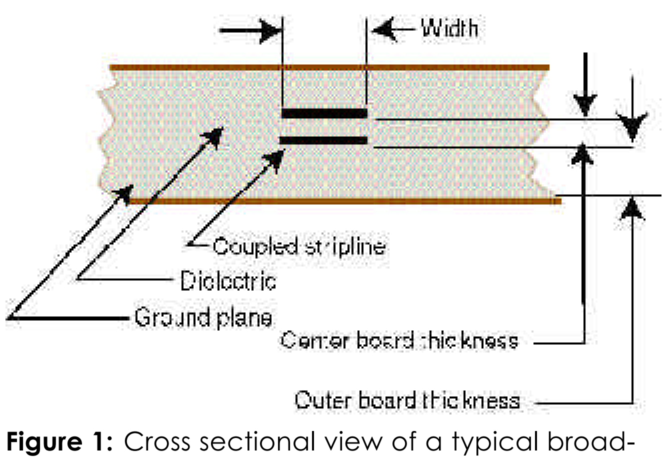

Figure 1: Cross sectional view of a typical broadside stripline coupler.

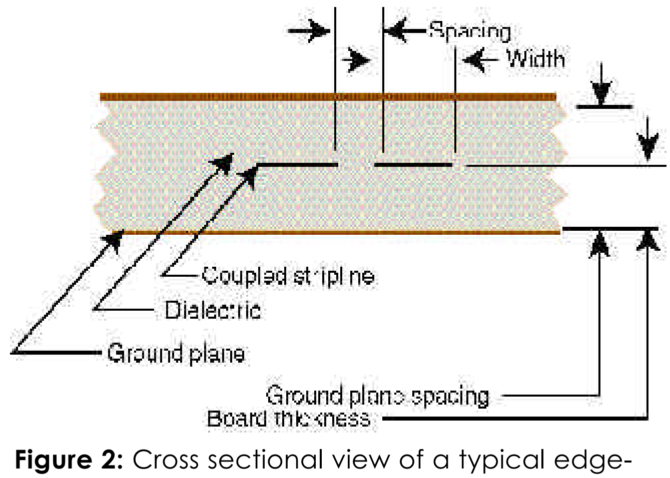

Figure 2: Cross sectional view of a typical edgewise stripline coupler.

Example designs for Broadside Coupled Lines

Substr.	outer	center	line	coupling	char.
type		board	board	width	coeff.
imped.				thk.(in.)	thk.(in.)
(in.) dB	Z_O ohm				
5880	0.031	0.005	0.200	1.47	9.83
5870	0.031	0.005	0.200	1.47	9.55
6002	0.030	0.005	0.060	1.88	23.53
6006	0.050	0.010	0.100	2.22	17.50
6010	0.050	0.010	0.125	2.11	11.39
TMM-3	0.060	0.015	0.200	2.42	17.36
TMM-4	0.060	0.015	0.200	2.42	14.80
TMM-6	0.050	0.015	0.200	2.76	11.81
TMM-10	0.050	0.015	0.175	2.82	10.68

Example designs for Edge Coupled Lines

Substr. type	board thk. (in.)	line spacing (in.)	line width (in.)	coupling coeff. dB	char. imped. Z_O ohm
5880	0.031	0.005	0.025	9.74	68.53
5870	0.031	0.005	0.025	9.74	66.59
6002	0.030	0.005	0.020	9.30	64.96
6006	0.025	0.005	0.015	9.65	47.77
6010	0.025	0.005	0.010	8.89	43.78
TMM-3	0.030	0.005	0.025	9.95	55.47
TMM-4	0.030	0.005	0.020	9.30	52.50
TMM-6	0.025	0.005	0.015	9.65	48.36
TMM-10	0.025	0.005	0.010	8.89	46.10

A. For broadside couplers

synthetic formulas are numbered as in reference 1.

Given

D = coupling coefficient in dB

Z_O = overall characteristic impedance in ohms

V = $e^{D/-8.68589}$ = voltage ratio

where

$$-8.68589 = \frac{-20}{\log_e(10)}$$

And knowing about the odd and even mode propagation

$$\frac{Z_{O,even}}{Z_{O,odd}} = \frac{1+V}{1-V}$$

$$Z_{O,even}\,Z_{O,odd} = Z_O^2$$

Solve for $Z_{O,even}$ and $Z_{O,odd}$

Solve [1] for k and use k to solve [2] for s

$$Z_{O,even} = \frac{188.3}{\sqrt{\varepsilon_r}}\;\frac{K(k')}{K(k)} \qquad [1]$$

where

s = ratio of line spacing to ground plane spacing

ε_r = relative permittivity or dielectric constant

where

K (k) = elliptic integral of the first kind with modulus k, not to be confused with the parameter m, the square of k. K(k) may be determined by a converging infinite series made to coverge rapidly by employing chain multiplication terms by alternate formula in 17.3.29 in reference 3

$$k' = \sqrt{1-k^2}$$

$$Z_{O,odd} = \frac{296.1s}{\sqrt{\varepsilon_r}\,\bullet\,\tanh^{-1}(k)} \qquad [2]$$

where

$\tanh^{-1}(i)$ = arc hyperbolic tangent of i

The ratio, w, of line width to ground plane spacing is given by [3].

$$w = \frac{2}{\pi}\tanh^{-1}(R) - s\tanh^{-1}\left(\frac{R}{k}\right) \qquad [3]$$

where

$$R = \sqrt{\frac{k-s}{\frac{1}{k}-s}}$$

B. For edgewise couplers analytic formulas are as in reference 2. Use w,s and b values to get values of Z_{O_e} and Z_{O_o} which are then used to get D and Z_O.

$$k_{even} = \tanh\left(\frac{\pi}{2}\,\frac{w}{b}\right)\tanh\left(\frac{\pi}{2}\,\frac{(w+s)}{b}\right) \qquad [3]$$

$$k'_{even} = \sqrt{1-k^2_{even}} \qquad [4]$$

$$Z_{O_{even}} = \frac{30\pi}{\sqrt{\varepsilon_r}}\;\frac{K(k'_{even})}{K(k_{even})} \qquad [2]$$

$$k_{odd} = \tanh\left(\frac{\pi}{2}\,\frac{w}{b}\right)\coth\left(\frac{\pi}{2}\,\frac{(w+s)}{b}\right) \qquad [6]$$

$$k'_{odd} = \sqrt{1-k^2_{odd}} \qquad [7]$$

$$Z_{O_{odd}} = \frac{30\pi}{\sqrt{\varepsilon_r}}\;\frac{K(k'_{odd})}{K(k_{odd})} \qquad [5]$$

where

w = width of lines

s = spacing between lines

b = ground plane spacing

tanh(i) = hyperbolic tangent of i

coth(i) = hyperbolic cotangent of i = 1/tanh(i)

$$Z_O = \sqrt{Z_{O_{even}}\,Z_{O_{odd}}}$$

$$D = -8.68589\log_e\frac{Z_{O_{even}}-Z_{O_{odd}}}{Z_{O_{even}}+Z_{O_{odd}}}$$

References:

1. S.B. Cohn, Characteristic Impedances for Broadside-Coupled Strip Transmission Lines, IRE Trans. MTT, Vol MTT-8, Nov. 1960, pp 633-637.
2. S.B. Cohn, Shielded Coupled-Strip Transmission Line, IRE Trans. MTT, Oct. 1955, pp 29-38.
3. M. Milne-Thompson, Section 17. Elliptic Integrals, NBS Handbook of Mathematical Functions, eds. M. Abramowitz, I.A. Stegun, June 1964.

Publication #